A Self-Consistent Event Biasing Scheme for Statistical Enhancement

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Statistical enhancement aims at reduction of the time necessary for computation of the desired device characteristics. Enhancement algorithms are especially useful when the device behavior is governed by rare events in the transport process. Such events are inherent for sub-threshold regime of device operation, simulations of effects due to discrete dopant distribution as well as tunneling phenomena. Virtually all Monte Carlo device simulators with statistical enhancement use population control techniques [1]. They are based on the heuristic idea for splitting of the particles entering given phase space region D of interest. The alternative idea - to enrich the statistic in D by biasing the probabilities associated with the transport of classical carriers - gives rise to the event-biasing approach. Due to the event biasing the behavior of the simulated numerical particles differs from that of Boltzmann carriers. Nevertheless the Boltzmann distribution function f is recovered by using the proper weights associated to the particles. The approach, first proposed for evolution of an initial condition [2], has been recently extended for stationary transport determined by boundary conditions [3]. Biased could be the events for particle evolution and/or the initial or boundary distributions. The approach is derived from the integral form of the linear Boltzmann equation (BE), where Coulomb interactions are neglected.

We generalize the approach for Hartree electrons in presence of both initial and boundary conditions. The BE becomes nonlinear via the Hartree component $\mathbf{E}(f)(\mathbf{r}, t)$ of the electric field. The steps used to derive event biasing can't be applied directly to BE, so that the solution is sought in the iterative procedure of coupling with the Poisson equation (PE). Employed is the fact that between two successive solutions of PE the electrical field is frozen. The BE is linear, and event biasing can be applied. The BE solution $f(t)$ is then obtained within a weighting scheme from the phase space position of the numerical particles. $f(t)$ gives the correct carrier density for the next solution of the PE. A key point in the proof is to use the Markovian character of the evolution: $f(t)$ becomes the initial condition f_0 of the BE for the next time step. Finally f_0 can be biased by inverting the weighting scheme used for $f(t)$. This means that the same particles, (with respect to phase space location and weight) which reach time t, can be used to continue the evolution in the next time interval. This proves that the biasing scheme can be used to provide the self-consistent distribution function at any time (t). In the thermodynamic limit $N \to \infty$ both Boltzmann and biased stochastic processes give the same evolution of the physical averages. For finite particle number N the computational efforts depend on the variance of the chosen stochastic process.

As an example we consider the sub-threshold regime at $T = 300K$ of a 15nm MOSFET [4]. Injection of particles with temperature $T > 300K$ biases the boundary conditions [3]. Such particles readily overcome the source potential barrier and enrich the statistics in the channel. A set of stochastic experiments with different N and T have been performed. For $N = 10^5$ the biased experiments recover precisely the physical averages and the self-consistent field. With decreasing N the behavior of the biased processes remains superior with respect to the unbiased counterparts. This is demonstrated in Fig. 1 for the evolution of the current density. Alternative biasing strategies are applied and the properties of the event-biasing scheme are analyzed.

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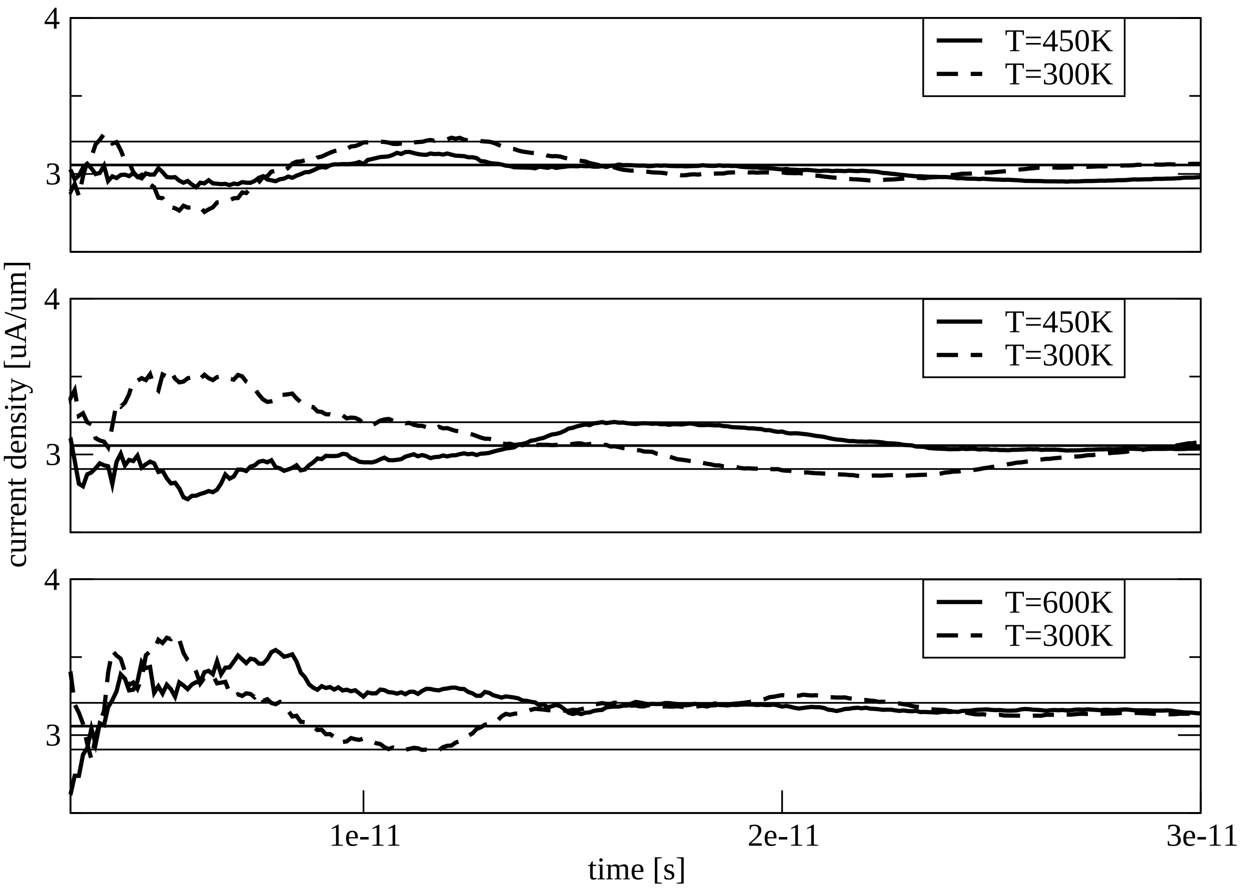

Figure 1: Top: $N = 10^5$. The $450K$ curve remains in the 5% limits about the mean value ($3.06\mu A/\mu m$) since the very beginning of the time plot ($3ps$ evolution time). Below: $N = 6.10^4$. The standard simulation, $T = 300K$, exhibits fluctuations well above $20ps$. Bottom:$N = 4.10^4$. The $T = 300K$ and $T = 600K$ curves show comparable variations. The reason is that more aggressive biasing causes a spread of the weights and thus increases the variance. Nevertheless the $T = 600K$ process demonstrates better stability for smaller number of particles, $N < 4.10^4$.

References

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